# Math Formulas: Complex numbers

## Definitions:

A complex number is written as $a + b\,i$ where $a$ and $b$ are real numbers an $i$, called the imaginary unit, has the property that $i^2 = -1$.

The complex numbers $z = a + b\,i$ and $\overline{z} = a - b\,i$ are called complex conjugate of each other.

## Formulas:

**Equality of complex numbers**

1. $$a + b\,i = c + d\,i \Longleftrightarrow a = c \;\; and \;\; b = d$$

**Addition of complex numbers**

2. $$(a + b\,i) + (c + d\,i) = (a + c) + (b + d)\,i$$

**Subtraction of complex numbers**

3. $$(a + b\,i) - (c + d, i) = (a - c) + (b - d)\,i$$

**Multiplication of complex numbers**

4. $$(a + b\,i) \cdot (c + d\,i) = (ac - bd) + (ad + bc)\,i$$

**Division of complex numbers**

5. $$\frac{a + b\,i}{c + d\,i} = \frac{a + b\,i}{c + d\,i} \cdot \frac{c - d\,i}{c - d\,i} = \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2}\,i$$

**Polar form of complex numbers**

6. $$a + b\,i = r \cdot (\cos\theta + i\,\sin\theta)$$

**Multiplication and division of complex numbers in polar form**

7. $$[r_1 (\cos\theta_1 + i \cdot \sin\theta_1)] \cdot [r_2 (\cos\theta_2 + i \cdot \sin\theta_2)] = r_1 \cdot r_2 [\cos(\theta_1 + \theta_2) + i \cdot \sin(\theta_1 + \theta_2)]$$

8. $$\frac{r_1 (\cos\theta_1 + i\,\sin\theta_1)}{r_2 (\cos\theta_2 + i\,\sin\theta_2)} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i \cdot \sin(\theta_1 - \theta_2)]$$

**De Moivre's theorem**

9. $$[r (\cos\theta + i\,\sin\theta)]^n = r^n (\cos(n\theta) + i\,\sin(n\theta))$$

**Roots of complex numbers**

10. $$[r (\cos\theta + i\,\sin\theta)]^{1/n} = r^{1/n} \left( \cos\frac{\theta + 2k\pi}{n} + i\,\sin\frac{\theta + 2k\pi}{n} \right) \quad k = 0, 1, \ldots, n - 1$$